Two-dimensional correction of data measured using a large pressure sensor

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Abstract

Various nozzle applications require description of a pressure impact field, which can mostly be done only by measurement. This paper describes how to obtain precise results using a real finite-size pressure sensor. In some cases, the impact distribution is very small or at least very narrow. Measuring such distributions with a pressure sensor, which is not small enough, results in incorrect data. The pressure is being averaged during the measurement and the shape of the obtained spray distribution is larger and the measured pressure maximum is lower than the real one. This paper describes how to correct the measured spray distribution. A two-dimensional Fast Fourier Transformation (FFT) is used to convert both the measured data and pressure sensor to the frequency domain. The correction of the measured spray distribution is done in the frequency domain and then an inverse two-dimensional FFT is used to convert the data back to the space domain. This allows suppressing noise in the measured data that would be extremely amplified. Very good results are presented using both the artificial and real data.

Nomenclature

\( A \) ............. Area, m\(^2\)  
\( f \) ............. Frequency, s\(^{-1}\), m\(^{-1}\)  
\( f_c \) ............. Nyquist critical frequency, s\(^{-1}\), m\(^{-1}\)  
\( G, H \) ........ Function in the frequency domain  
\( g, h \) ........ Function in the space or time domain  
\( i \) ............. Imaginary number  
\( N, n \) ........ Number of samples  
\( p \) ............. Pressure, Pa  
\( S \) ............. Function of the sharpened data in the frequency domain  
\( t \) ............. Time, s  
\( x, y \) ........ Cartesian coordinates, m  
\( \Delta \) ............. Time interval, s
1 Introduction

Measuring the pressure distribution, a pressure sensor of a finite size has to be used. The precision of the measured data depends on the size ratio of a nozzle spray spot and the sensor. As the ratio becomes smaller the precision of the measured data is getting worse because of averaging the impact forces. An inverse algorithm can be used to compute a real pressure distribution using the measured data. This method converts data to the frequency domain, creates an inverse pressure sensor, multiplies the data measured using the inverse pressure sensor and converts the result back to the space domain. Although this method works well also with noisy data, a limit of this method exists.

2 Measurement of pressure distribution

Measuring the pressure distribution, the nozzle sprays on a moving plate (see Figure 1). This plate is equipped with a pressure sensor that may be of circular or rectangular shape, and its dimensions range from 1.5 mm to 3 mm. For a given nozzle configuration, a pressure is measured as position dependent while the plate with the sensor is moving under the spraying nozzle.

For nozzles with a very narrow spray spot and for a small distance of a nozzle from the moving plate, the measured data doesn’t represent a real pressure distribution. The values are averaged over a “large” pressure sensor. To correct the measured data, a Fourier transform is used.

3 Fourier transform

A physical process can be described either in the time domain, by the values of

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Figure 1: Pressure distribution measurement.
some quantity \( h \) as a function of time \( t \), e.g., \( h(t) \), or else in the frequency domain \([1]\), where the process is specified by giving its amplitude \( H \) as a function of frequency \( f \). For many purposes it is useful to think of \( h(t) \) and \( H(f) \) as being two different representations of the same function. One goes back and forth between these two representations by means of Fourier transform equations

\[
H(f) = \int_{-\infty}^{\infty} h(t) e^{2\pi i ft} \, dt \tag{1}
\]

\[
h(t) = \int_{-\infty}^{\infty} H(f) e^{-2\pi i ft} \, df . \tag{2}
\]

If \( t \) is measured in seconds, then \( f \) in eqns (1, 2) is in cycles per second, or Hertz (the unit of frequency). However, the equations work with other units, too. If \( h \) is a function of position \( x \) (in meters), \( H \) will be a function of inverse wavelength (cycles per meter), and so on.

### 3.1 Convolution Theorem

With two functions \( h(t) \) and \( g(t) \), and their corresponding Fourier transforms \( H(f) \) and \( G(f) \), we can form two combinations of special interest. The convolution of the two functions, denoted \( g \cdot h \), is defined by

\[
g \cdot h = \int_{-\infty}^{\infty} g(\tau) \cdot h(t - \tau) \, d\tau . \tag{3}
\]

Note that \( g \cdot h \) is a function in the time domain and that \( g \cdot h = h \cdot g \). It turns out that the function \( g \cdot h \) is one member of a simple transform pair

\[
g \cdot h \leftrightarrow G(f) \cdot H(f) . \quad \text{“Convolution Theorem”} \tag{4}
\]

In other words, the Fourier transform of the convolution is just the product of the individual Fourier transforms \([1]\).

### 3.2 Fourier transform of discretely sampled data

In the most common situations, function \( h(t) \) is sampled (i.e., its value is recorded) at evenly spaced intervals in time. Let \( \Delta t \) denote the time interval between consecutive samples, so that the sequence of sampled values is

\[
h_n = h(n \Delta t) \quad \text{for} \quad n = ..., -3, -2, -1, 0, 1, 2, 3, ...
\]

(5)

The reciprocal of the time interval \( \Delta t \) is called the sampling rate; if \( \Delta t \) is measured in meters, for example, then the sampling rate is the number of samples recorded per meter.
3.3 Sampling theorem and aliasing

For any sampling interval $\Delta t$, there is also a special frequency $f_c$, called the *Nyquist critical frequency* [1], given by

$$f_c = \frac{1}{2\Delta t}.$$  

(6)

The Nyquist critical frequency is important for two related, but distinct, reasons. The first one is the remarkable fact known as the sampling theorem: If a continuous function $h(t)$, sampled at an interval $\Delta t$, happens to be bandwidth limited to frequencies smaller in magnitude than $f_c$, i.e., if $H(f) = 0$ for all $|f| \geq f_c$, then the function $h(t)$ is completely determined by its samples $h_n$.

This is a remarkable theorem for many reasons, among them that it shows that the “information content” of a bandwidth limited function is, in some sense, infinitely smaller than that of a general continuous function.

The bad news concerns the effect of sampling a continuous function that is not bandwidth limited to less than the Nyquist critical frequency. In that case, it turns out that all of the power spectral density that lies outside of the frequency range $-f_c < f < f_c$ is spuriously moved into that range. This phenomenon is called aliasing. Any frequency component outside of the frequency range ($-f_c; f_c$) is aliased (falsely translated) into that range by the very act of discrete sampling (see Figure 2).

4 Correction of measured data

As mentioned in previous sections, the measured distribution differs from the real one. An example of simulated real and measured distribution of pressure is shown in Figure 3. The measured maximum is 77.8 MPa but the real one is 100 MPa. The measured impact shape is wider than the real one.

An impact was 20x80 mm and a sensor with circular active surface was assumed. The diameter active surface was 12 mm. Such a large sensor averages

![Figure 2: Fourier transform of sampled function is defined only between plus and minus of the Nyquist critical frequency. Power outside that range is folded over or “aliased” into the range [1].](image)
values and one measured value is equal to
\[ p = \frac{1}{A} \iint_A p(x, y) \, dx \, dy \quad (7) \]
where \( A \) is the surface of the sensor. The whole measured distribution can be described using the following convolution equation
\[ g \cdot h = \iint_A g(x, y) \cdot h(X - x, Y - y) \, dx \, dy \quad (8) \]
where \( h \) is a filter function. This filter function describes how the sensor averages real values.

To obtain a real distribution from a measured one, a convolution equation can also be used. In this case, the filter function is an inverse function to the sensor filter function. The convolution, inverse function computation and noise reduction can be done more easily in the frequency domain than in the space domain.

### 4.1 Conversion to frequency domain

Let us assume the data measured using the circular sensor (Ø12 mm) as shown

Figure 3: An example of (a) simulated real distribution and (b) simulated measured distribution using a circular sensor of 12 mm in diameter.

Figure 4: (a) Data measured in frequency domain; (b) circular sensor in frequency domain.
in Figure 3. Measured data and sensor filter function are transformed from the space domain into the frequency domain using FFT (Fast Fourier Transform). The transformed values are shown in Figure 4 where the amplitude axes use a logarithmic scale.

4.2 Inverse Sensor Function and Data Sharpening

As we have values transformed into the frequency domain we can easily compute the inverse sensor filter using

$$ h^{-1}(f_x, f_y) = \frac{1}{h(f_x, f_y)} $$

(9)

The inverse sensor filter in the frequency domain is shown in Figure 5a. Having the inverse filter we can do the convolution, eqn (4), in the frequency domain using this inverse filter and measured data to obtain a real pressure distribution (still in the frequency domain), see Figure 5b. In our case, the convolution is described by

$$ S(f_x, f_y) = G(f_x, f_y) \cdot H^{-1}(f_x, f_y) $$

(10)

where $G$ are measured data, $H^{-1}$ is the inverse sensor function and $S$ represents a sharpened data. Transforming sharpened data from the frequency domain into

![Figure 5](https://via.placeholder.com/150)

**Figure 5:** (a) Inverse sensor function in frequency domain; (b) convolution of measured data and inverse sensor function in frequency domain; (c) sharpened measured data - convolution of measured data and inverse sensor function in frequency domain; (d) real pressure distribution in frequency domain.
the space domain using inverse FFT we obtain a pressure distribution which should be very close to the real pressure distribution (see Figure 5c). As you can see, some noise is visible in the sharpened data (small waves). Noise can be partially suppressed as described in the following section.

4.3 Noise reduction and aliasing phenomenon

Noise is most significant at high frequencies and because we are working in the frequency domain, noise can be suppressed. As you can see in Figure 4b, the sensor function consists of the main frequency spectrum (the highest peak in the middle) and higher harmonic frequencies (the other waves). Cutting off the higher harmonic frequencies and making them equal to zero also in the inverse sensor filter, we get a cut inverse sensor filter. Using this filter for convolution (Eq. 10) instead of the inverse filter (shown in Figure 8a) and transforming sharpened data using inverse FFT, we get sharpened measured data with suppressed noise. The computed maximum is not 100 MPa but only 96.3 MPa. This is due to the aliasing effect that is described in previous sections.

The sampling continuous function that is not bandwidth limited to less than the Nyquist critical frequency results in an incorrect frequency spectrum. Any frequency component that lies outside of the range \( f < f_c \) is spuriously moved into that range (this phenomenon is called aliasing). This effect is more

![Figure 6:](a) Circular sensor function in space domain filtered using low-pass filter; (b) circular sensor function in frequency domain with removed high frequency; (c) cut inverse sensor function in frequency domain; (d) sharpened measured data.)
significant for the sensor function because of sharp edges of the sensor.

All we can do to avoid aliasing is to use a low-pass filter. The sensor function passes through the Gaussian low-pass filter in the space domain. A smoothed sensor function is shown in Figure 6a. Transforming this function into the frequency domain (see Figure 6b), we get high frequencies equal to zero. This means there is no aliasing effect.

Using the cut smooth inverse sensor filter for convolution (see Figure 6c) and transforming sharpened data using inverse FFT, we get sharpened measured data (see Figure 6d) with a maximum of 98.9 MPa which is very close to the real maximum 100 MPa. You can also notice that the noise is well suppressed compared with the computed result shown in Figure 5c.

5 Sharpening of real measured data

Real measured data are shown in Figure 7a. The measurement was made for high-pressure flat jet nozzle where the distance from the surface was 150 mm and water pressure was 20 MPa. The measurement was made using a square sensor (3x3 mm). Some noise in measured data is obvious. The maximum in sharpened data (see Figure 7b) rose about 18% (from 1534 to 1814 kPa).

5 Figure 7: (a) Real data measured using rectangular sensor (3x3 mm); (b) sharpened data

Figure 8: (a) Real data measured using circular sensor (1.5 mm in diameter); (b) sharpened data
Another measurement was made using a different measuring apparatus. A circular sensor of 1.5 mm in diameter was used. Two “overlapping” flat nozzles were measured (see Figure 8). The measured data are almost noise free compared to the previous data measured using a rectangular sensor. The measuring area was 30x160 mm. The rest of the area shown in Figure 8a, where the value is equal to zero, is necessary only for FFT. The maximum of sharpened data rose from 1527 to 1587 kPa. The increase is small because a small pressure sensor was used.

6 Conclusion

As shown, any measured data are biased due to the finite size of the measuring sensor. The distortion becomes higher as the size ratio of a nozzle spray spot and the sensor decreases. An inverse method that computes real pressure distribution from measured data has been presented. The presented method works well with any shape of the measuring sensor.

Acknowledgement

Both the theoretical and experimental parts of the work were supported by the grant contract no. 106/02/1571 of the Grant Agency of the Czech Republic.

References